

Algorithmic Game Theory

Auction theory in practice

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Allocation rules and truthful mechanisms

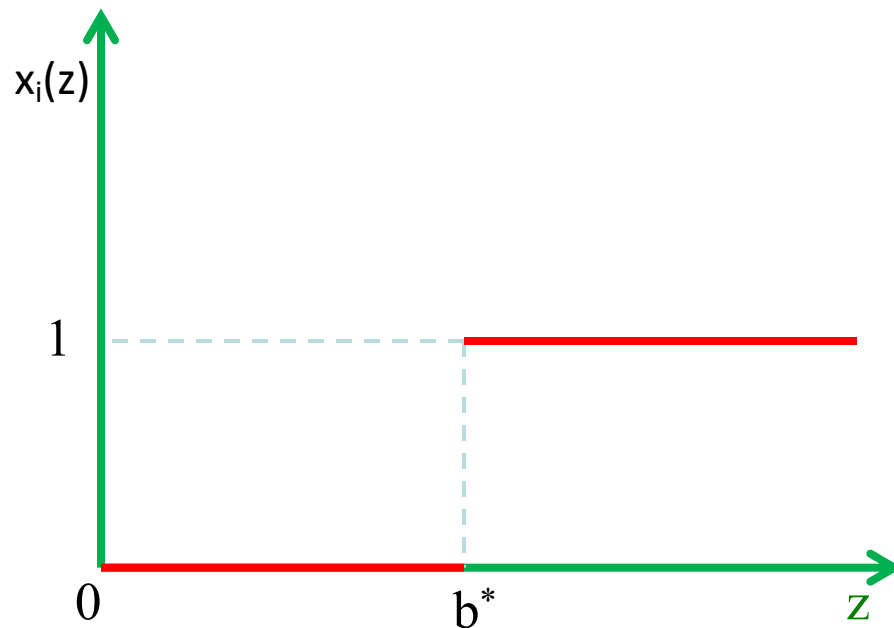
- We recall first some definitions we saw in previous lectures
- Consider a mechanism with allocation rule \mathbf{x}
- Definition: An allocation rule is **monotone** if for every i , and every profile \mathbf{b}_{-i} , the allocation $x_i(z, \mathbf{b}_{-i})$ to i is non-decreasing in z
 - i.e., bidding higher can only get you more stuff

[Myerson '81]

- Theorem: For every single-parameter environment,
 - An allocation rule \mathbf{x} can be turned into a truthful mechanism if and only if it is monotone
 - If \mathbf{x} is monotone, then there is a unique payment rule \mathbf{p} , so that (\mathbf{x}, \mathbf{p}) is a truthful mechanism

Myerson's lemma and payment formula

- For the payment rule, we need to look for each bidder at the allocation function $x_i(z, \mathbf{b}_{-i})$
- For the single-item truthful auction:
 - Fix \mathbf{b}_{-i} and let $b^* = \max_{j \neq i} b_j$



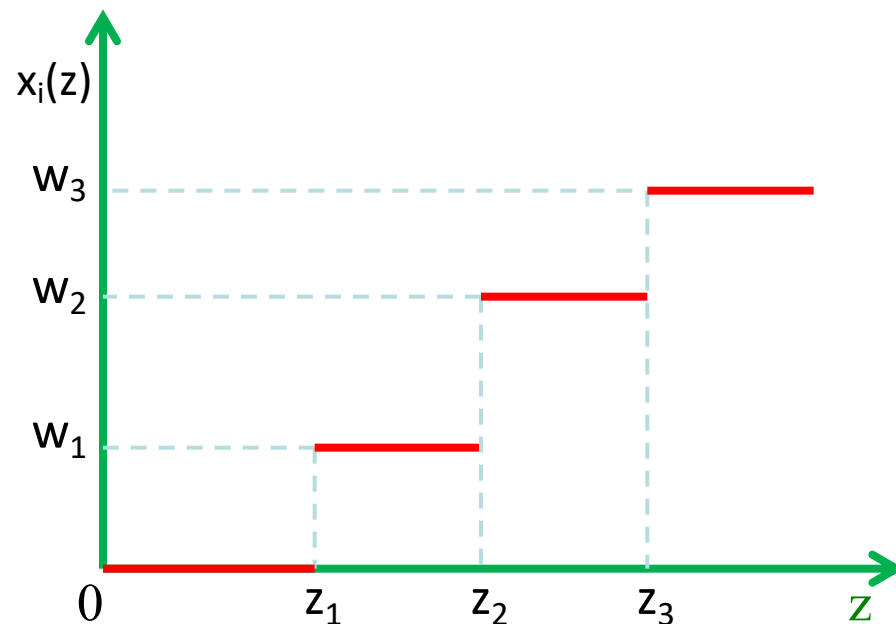
Facts:

- For any fixed \mathbf{b}_{-i} , the allocation function is piecewise linear with 1 jump
- The Vickrey payment is precisely the value at which the jump happens
- The jump changes the allocation from 0 to 1 unit

Myerson's lemma and payment formula

For most scenarios of interest

- The allocation is piecewise linear with multiple jumps
- The jump determines how many extra units the bidder wins

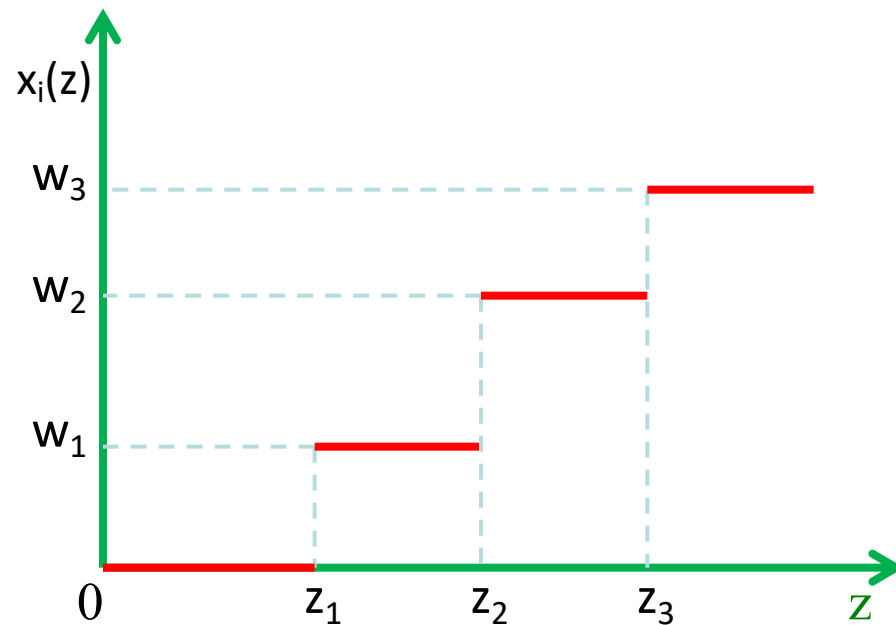


- Suppose bidder i bids b_i
- Look at the jumps of $x_i(z, b_i)$ in the interval $[0, b_i]$
- Suppose we have k jumps
- Jump at $z_1 = w_1$
- Jump at $z_2 = w_2 - w_1$
- Jump at $z_3 = w_3 - w_2$
- ...
- Jump at $z_k = w_k - w_{k-1}$

Myerson's lemma and payment formula

For most scenarios of interest

- The allocation is piecewise linear with multiple jumps
- The jump determines how many extra units the bidder wins



Payment formula

- For each bidder i at a profile b , find all the jump points within $[0, b_i]$
- $$p_i(b) = \sum_j z_j \cdot [\text{jump at } z_j]$$
$$= \sum_j z_j \cdot [w_j - w_{j-1}]$$
- The formula can also be generalized for monotone but not piecewise linear functions

Sponsored Search Auctions

What is sponsored search?

Google™ Αναζήτηση [Σύνθετη Αναζήτηση](#) [Προτιμήσεις](#)

Αναζήτηση: παγκόσμιος ιστός σελίδες στα Ελληνικά σελίδες από Ελλάδα

Παγκόσμιος ιστός Αποτελέσματα 1 - 10 από περίπου 4.140.000 για **crm software**. (0,17 δευτερόλεπτα)

Interworks Web CRM
www.interworks.gr Το πρώτο Web CRM στην Ελλάδα. Διακρίμαστε το δωρεάν για 30 ημέρες

Goldmine CRM
www.alexandermoore.com Αυξήστε τις Πωλήσεις με το Νο1 CRM στις ΗΠΑ & 10 χρόνια στην Ελλάδα

Crm Software
www.CRMdesk.com Web-based Help Desk, Customer Service and Online Support Software

Συμβουλή: [Αναζήτηση αποτελεσμάτων μόνο σε Ελληνικά](#). Μπορείτε να επιλέξετε τη γλώσσα αναζήτησης στη σελίδα [Προτιμήσεις](#)

[Διαχείριση πελατολογίου, Συναλλαγών, Πελατών, Πελατολόγιο ...](#)
Greek **CRM software**, database **software**, ΕΣΟΔΑ, ΕΞΟΔΑ, ... **crm network fax software**, καταχώρηση τιμολογίων, πρόγραμμα πελατων, εσοδα εξοδα, κεφαλαιο, ...
www.starmessage.gr/crm_software.html - 66k - Προσωρινά αποθηκευμένη - Παρόμοιες σελίδες

[CRM Software, Customer Relationship Management, CRM Solutions from ...](#) - [[Μετάφραση αυτής της σελίδας](#)]
CRM from Oncontact. Your source for customer relationship management or CRM software, CRM solutions and customer relationship management software.
www.oncontact.com/ - 12k - Προσωρινά αποθηκευμένη - Παρόμοιες σελίδες

[CRM SOFTWARE - SalesManager Hellas - Customer Relationship ...](#)
Μία από τις πλέον σύγχρονες τάσεις της επιχειρηματικότητας αφορά στην « Διαχείριση των Σχέσεων με τους Πελάτες / Customer Relationship Management» ή « CRM». ...

Advertising slots

Σύνδεσμοι διαφημιζομένων

AuraPortal: BPMS with CRM
5 in 1: Process, CRM/E-Business, Intranet, Documents, ECM Portals
www.AuraPortal.com

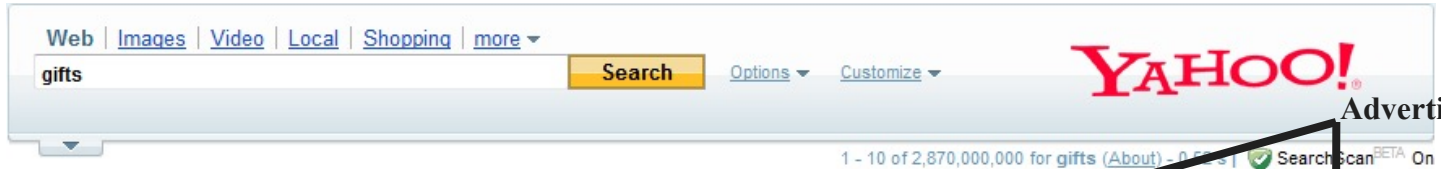
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EasyConsole eCRM
Σύστημα Διαχείρισης Πελατών (CRM) για Μικρές και Μεγάλες επιχειρήσεις
www.dynamicworks.eu

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What is sponsored search?



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CWDesignLLC.com - great collection of gold, silver and crystal purse h...
www.cwdesignllc.com

Gift Ideas: Christmas Gifts, Anniversary And Birthday
Gift ideas from popular to unique. Gifts.com's gift experts find great gifts for every occasion, including gift ideas for wedding presents, birthdays and anniversary.
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A community site where consumers share their best gift ideas with each other, organized into categories like unusual ... recommends creative gifts and shows ...
www.surprise.com

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American Metal Arts
Custom made Metal Art Hand Crafted and Made in the USA.
www.AmericanMetalArtsStudios.com

Wedding Gifts
Create unique gifts with your photos. T-shirts, mugs, postage & more.
www.Zazzle.com

Collectible Stock Certificates
We buy and sell collectible stock certificates. Old stock certificates...
www.giftofhistory.com

How does it work?

- For a fixed search term (e.g. *ipod*)
 - n advertisers
 - k slots (typically $k \ll n$)
 - An auction is run for every single search
 - Each advertiser (bidder) is interested in getting himself displayed in one of the slots
 - And usually they prefer a slot as high up as possible
 - Same auction is also run for related keywords (e.g. “buy ipod”, “cheap ipod”, “ipod purchase”, ...)
 - The advertiser can determine for which phrases to participate

How does it work?

- Bidders submit an initial budget which they can refresh weekly or monthly
- Bidders also submit an initial bid which they can adjust as often as they wish
- The auction selects the winners to be displayed
- Different charging models exist: **Pay Per Click, Pay Per Impression, Pay Per Transaction**
- Currently, most popular is Pay Per Click
- A bidder is charged only if someone clicks on the bidder's ad

The Actors

- The Search engine:
 - Wants to make as much revenue as possible
 - At the same time, wants to make sure users receive meaningful ads and bidders do not feel that they were overcharged
 - Big percentage of Google's revenue has been due to these auctions!
- The Bidders:
 - Want to occupy a high slot and pay as little as possible
- The Searchers:
 - Want to find the most relevant ads with respect to what they are looking for

Analyzing sponsored search auctions

- We will focus on the bidders' side
- Model parameters for each bidder i
 - Private information: v_i = maximum amount willing to pay per click = value/happiness derived from a click (private information)
 - Each bidder i submits a bid b_i for willingness to pay per click (b_i may differ from v_i)
 - We will ignore the budget parameter
 - In many cases, it is large enough and cannot affect the game
 - Hence, we have a single-parameter problem

Analyzing sponsored search auctions

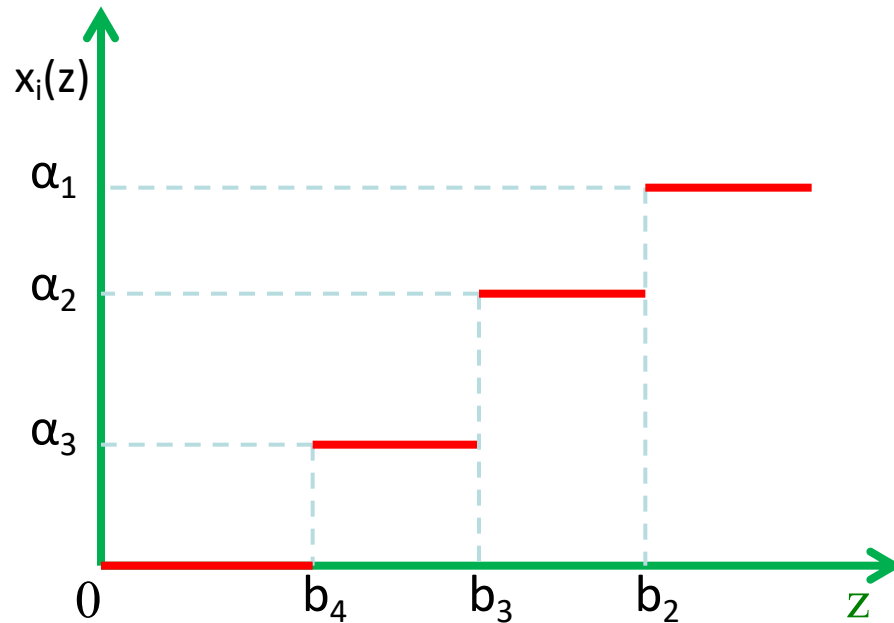
- We will focus on the bidders' side
- Model parameters for each slot j
 - α_j = Click-through rate (CTR) of slot j = probability that a user will click on slot j
 - Assume it is independent of who occupies slot j
 - We can generalize to the case where the rates are weighted by a quality score of the advertiser who takes each slot
 - The search engines update regularly the CTRs and statistics show that
$$\alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \dots \geq \alpha_k$$
 - Users tend to click on higher slots
 - Validation also by eye-tracking experiments

Analyzing sponsored search auctions

- How shall we allocate the k slots to the n bidders?
- **Most natural allocation rule:** for $i=1$ to k , give to the i -th highest bidder the i -th best slot in terms of CTR
 - Remaining $n-k$ bidders do not win anything
- For convenience, assume that $b_1 \geq b_2 \geq b_3 \geq \dots \geq b_n$
- Expected value of a winning bidder i : $\alpha_i v_i$
- Is this rule monotone?
- Yes, bidding higher can only get you a better slot
- Hence we can apply Myerson's formula to find the payment rule
- For each bidder i , let $x_i(b_i, b_{-i}) \in \{0, \alpha_k, \alpha_{k-1}, \dots, \alpha_1\}$

Myerson's lemma for sponsored search auctions

- Let's analyze the highest bidder with bid b_1
- Suppose we have 3 slots and $n > 3$ bidders



- Look at the jumps of x_i in the interval $[0, b_1]$
- Jump at $b_4 = \alpha_3$
- Jump at $b_3 = \alpha_2 - \alpha_3$
- Jump at $b_2 = \alpha_1 - \alpha_2$

Total payment:

$$b_4 \alpha_3 + b_3 (\alpha_2 - \alpha_3) + b_2 (\alpha_1 - \alpha_2)$$

Myerson's lemma for sponsored search auctions

- More generally, for the i -th highest bidder, there will be $k-i+1$ jumps

$$p_i(\mathbf{b}) = \sum_{j=i}^k b_{j+1} [\alpha_j - \alpha_{j+1}]$$

- Under pay-per-click, no actual payment takes place at the end of every auction, unless there is a click by a user
- Need to scale so that expected per-click payment is $p_i(\mathbf{b})$
- Proposed per-click payment to bidder in i -th slot: $p_i(\mathbf{b})/\alpha_i$
- By Myerson, no other payment can achieve truthfulness with the same allocation rule

Sponsored search auctions in practice

- In practice most engines do not use the payment of Myerson's lemma
- But they use the same allocation rule
- **The Generalized Second Price Mechanism (GSP)** - initial version:
 - The search engine ranks the bids in decreasing order:
 $b_1 \geq b_2 \geq \dots \geq b_n$
 - The i -th highest bidder takes the i -th best slot
 - Every time there is a click on slot i , bidder i pays b_{i+1}
 - There is also a reserve price (opening bid), initially the same for every keyword (\$0.1), later became keyword-dependent

The Generalized Second Price Mechanism (GSP)

- A better version:
 - The search engine keeps a quality score q_i for each bidder i
 - Yahoo, Bing (till a few years ago): q_i is the click-through rate of i (probability of a user clicking on an ad of bidder i)
 - Google: q_i depends on click-through rate, relevance of text and other factors
 - The search engine ranking is in decreasing order of $q_i \times b_i$
 $q_1 \times b_1 \geq q_2 \times b_2 \geq \dots \geq q_n \times b_n$
 - The first k bidders of the ranking are displayed in the k slots
 - Every time there is a click on slot i , bidder i pays the minimum bid required to keep his position, i.e. $(q_{i+1} \times b_{i+1}) / q_i$

The Generalized Second Price Mechanism (GSP)

- Myerson's lemma implies GSP cannot be truthful
 - Otherwise, its payment rule would coincide with the Myerson formula
- The deployment of GSP was probably just an educated guess
 - As an attempt to generalize the Vickrey auction and use something simple that looked close to truthful!
- Nevertheless...
 - For a long period, revenue from GSP was 95% of Google's revenue
 - Still nowadays an important percentage of search engines' revenue

Multi-unit auctions

Multi-unit Auctions

Auctions for selling multiple identical units of a **single good**

In practice:

- US Treasury notes, bonds
- UK electricity auctions (output of generators)
- Carbon allowance markets (pollution rights)
- Spectrum licences
- Various online sales

Multi-unit Auctions

Online sites offering multi-unit auctions

- US
 - www.onlineauction.com
- UK
 - uk.ebid.net
- Greece
 - www.ricardo.gr
 - Actually not any more...
- ...

Some Notation

- n bidders
- k available units of an indivisible good
- Bidder i has valuation function $v_i : \{0, 1, \dots, k\} \rightarrow \mathbb{R}$
 - $v_i(j)$ = value of bidder i for obtaining j units
- Representation with marginal valuations:
 - $m_i(j) = v_i(j) - v_i(j-1)$ = additional value for obtaining the j -th unit, if already given $j-1$ units
 - $(m_i(1), m_i(2), \dots, m_i(k))$: vector of marginal values

Submodular Valuations

- In the multi-unit setting, a valuation v_i is *submodular* iff
$$\forall x \leq y, \quad v_i(x + 1) - v_i(x) \geq v_i(y + 1) - v_i(y)$$
 - Hence: $m_i(1) \geq m_i(2) \geq \dots \geq m_i(k)$ (decreasing marginal values)
- In many multi-unit auctions, bidders are asked to submit a submodular valuation
 - Makes sense due to the saturation of getting more and more units
- **Valuation compression:** Even if bidders are not submodular, they would still have to express their preferences by a submodular function

A Bidding Format for Multi-unit Auctions

- Used in various multi-unit auctions

[Krishna ' 02, Ch. 12-13, Milgrom ' 04, Ch. 7]

1. The auctioneer asks each bidder to submit a vector of decreasing marginal bids
 - $b_i = (b_i(1), b_i(2), \dots, b_i(k))$
 - $b_i(1) \geq b_i(2) \geq \dots \geq b_i(k)$
2. The bids are ranked in decreasing order and the k highest win the units

Simplified format in some cases: Uniform bidding, i.e., ask for a bid per unit + number of units demanded

Example



$$\mathbf{b}_1 = (45, 42, 31, 22, 15)$$

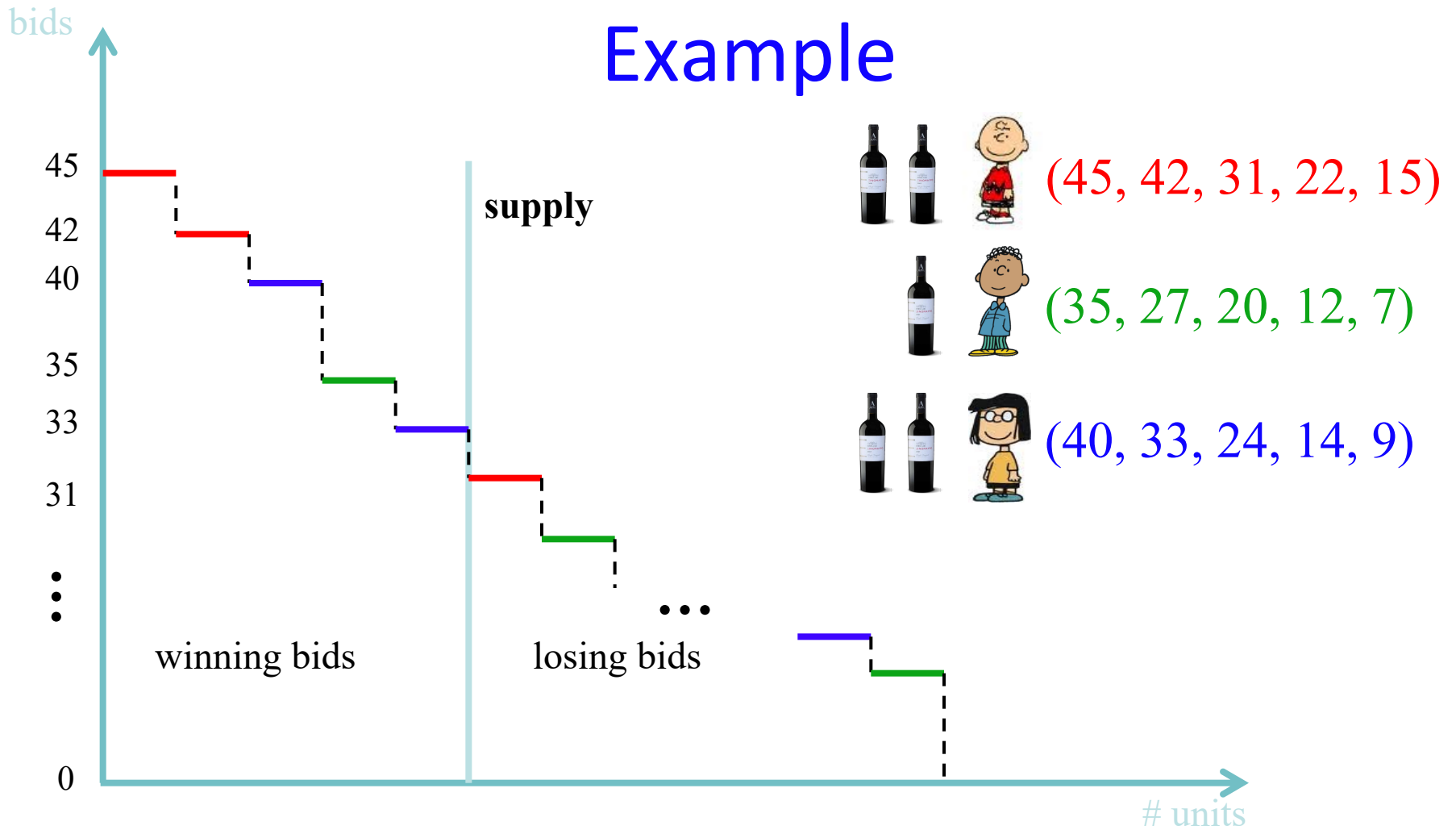


$$\mathbf{b}_2 = (35, 27, 20, 12, 7)$$



$$\mathbf{b}_3 = (40, 33, 24, 14, 9)$$

Example



How should we charge the winners?

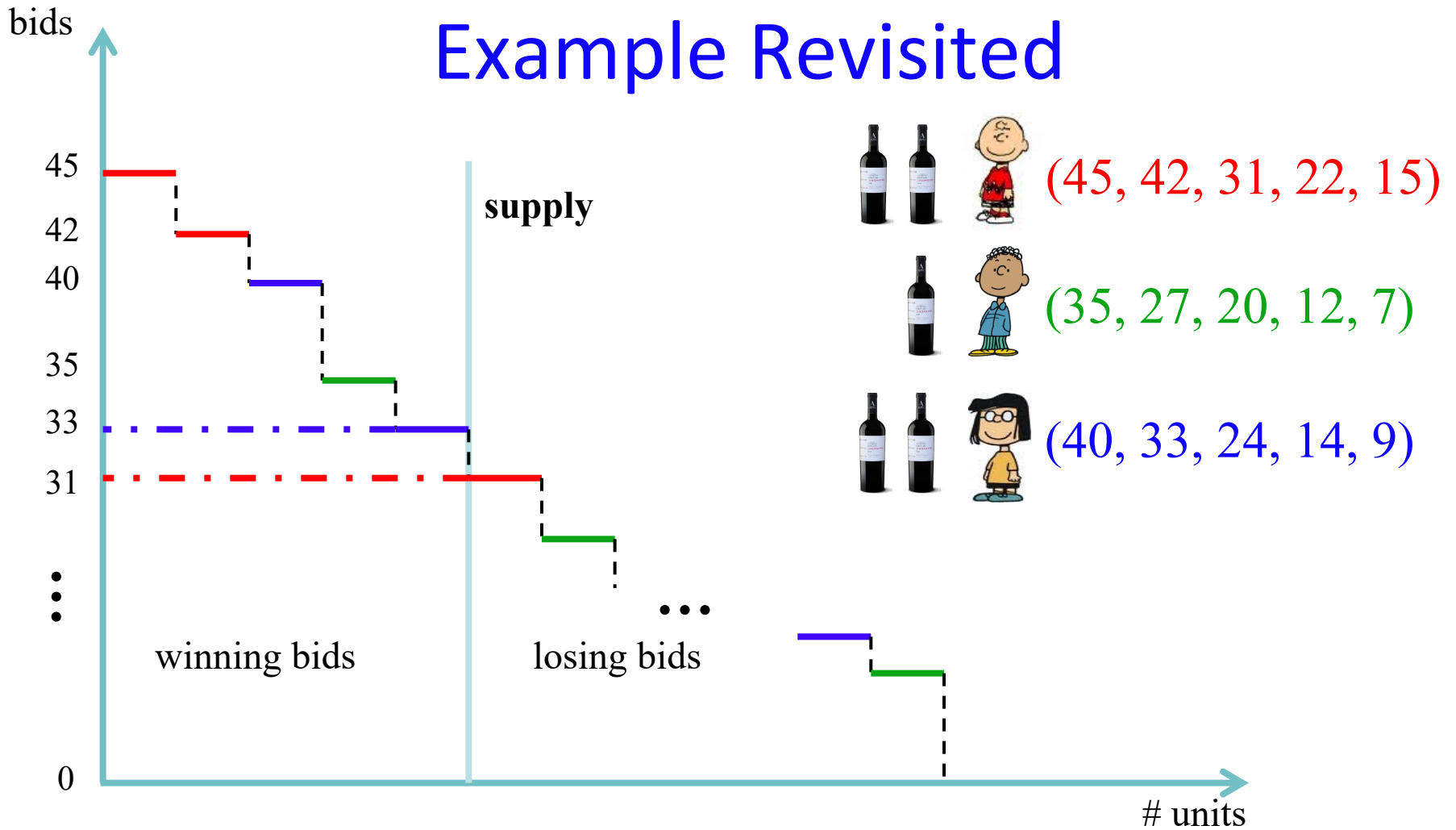
Pricing Rules

1. Multi-unit Vickrey auction (VCG) [Vickrey '61]
 - Each bidder pays the externality he causes to the others
 - Generalization of single-item 2nd price auction
 - Good theoretical properties, truthful, but barely used in practice
2. Discriminatory Price Auction (DPA)
 - Bidders pay their bids for the units won
 - Generalization of 1st price auction
 - Not truthful, but widely used in practice

Pricing Rules (cont' d)

3. Uniform Price Auction (UPA) [Friedman 1960]
 - Same price for every unit
 - Interval of prices to pick from:
 - [highest losing bid, lowest winning bid]
 - This lecture: price = highest losing bid
 - For 1 unit, same as Vickrey auction
 - For ≥ 2 units, not truthful, but widely used in practice (following the campaign of Miller and Friedman in the 90' s)

Example Revisited



Interval of candidate prices for UPA = [31, 33]

Uniform price = 31

Uniform Price vs Discriminatory?

- Debate still going on for treasury auctions
- DPA is thought to raise more revenue (no formal justification though)
- UPA eliminates complaints arising from price discrimination (identical goods should cost the same!)

Equilibrium analysis of non-truthful mechanisms

Non-truthful mechanisms

- As already seen, there are plenty of settings where the mechanism employed is not truthful
 - Sponsored search
 - Auctions for government bonds
 - Some types of auctions for telecom/spectrum licences (e.g., core-selecting auctions)
- Why?
 - Low revenue often achieved by truthful auctions, e.g., by VCG
 - Complexity: Social welfare maximization may turn out too difficult to solve (which is a required step in VCG-based mechanisms)
- **[Ausubel, Milgrom '06]:** The lovely but lonely Vickrey auction
 - Chapter 1 in the book “Combinatorial Auctions”

Non-truthful mechanisms

- How do we evaluate non-truthful mechanisms?
 - If the bidders are non-truthful, can we argue about the social welfare generated?
- We can think of the equilibria as the most likely outcomes to occur
 - If these games are played frequently, players may end up at an equilibrium by adjusting gradually their strategies
 - Thus, we can take the social welfare or revenue achieved at an equilibrium as an evaluation metric

PoA in auctions

- Consider an auction where v_i = actual valuation function of bidder i
 - It can be either single or multi-parameter
- Let \mathbf{b} be a pure Nash equilibrium with resulting allocation: $(x_1, \dots, x_n) = (x_1(\mathbf{b}), \dots, x_n(\mathbf{b}))$
- Social Welfare at \mathbf{b} : $SW(\mathbf{b}) = \sum v_i(x_i)$
- OPT = Optimal welfare (as determined by the valuations)

$$PoA = \sup_{\mathbf{b}} OPT/SW(\mathbf{b})$$

Where the supremum can be either over all pure or over all mixed equilibria

PoA in sponsored search auctions

- PoA can become unbounded in worst case
- [Lahaie '06]: $\text{PoA} \leq (\min_{1 \leq i \leq k-1} \min\{\alpha_{i+1}/\alpha_i, 1 - (\alpha_{i+2}/\alpha_{i+1})\})^{-1}$
 - For pure equilibria, when we have $k \geq 2$ slots
 - Where recall α_i is the CTR of slot i , and assume $\alpha_{k+1} = 0$
- For arbitrary auctions, the ratios of the CTRs can become arbitrarily high
- In some cases, the click data fit well with an exponential decay model (geometric CTRs): $\alpha_i \propto 1/\delta^i$ for a constant δ
 - [Feng, Bhargava, Pennock '07]: $\delta = 1.428$ using various empirical datasets
 - In these cases, $\text{PoA} \leq (\min\{1/\delta, 1-1/\delta\})^{-1}$
 - Hence, low inefficiency under geometric CTRs

PoA in sponsored search auctions

- One can also study PoA under restrictions on the set of equilibria under consideration
- E.g., some “bad” equilibria arise when some players overbid and at the same time some high-valued players underbid
- **The no-overbidding assumption:** Focus on equilibria where $b_i \leq v_i$
 - Such bidders are also referred to as conservative bidders
 - Initiated in [Christodoulou, Kovacs, Schapira '08], and assumed in several follow up works
- Can PoA be better under no-overbidding?

PoA in sponsored search auctions


- [Paes Leme, Tardos '10]: Under no-overbidding
 - $\text{PoA} \leq 1.618 (= \phi)$ for pure equilibria
 - $\text{PoA} \leq 4$ for mixed equilibria
- [Lucier, Paes Leme '11, Caragiannis et al. '11, '15]:
Currently best known:
 - $\text{PoA} \leq 1.28$ for pure equilibria
 - $\text{PoA} \leq 2.31$ for mixed equilibria
- For lower bounds, it is known that $\text{PoA} \geq 1.259$
- **Main conclusion:** For conservative bidders, selfish behavior does not lead to socially bad outcomes


PoA in multi-unit auctions


- A PoA analysis can be carried out for any other non-truthful auction
- For multi-unit auctions, PoA can be affected by the phenomenon of “demand reduction”
 - [Ausubel, Cramton '96]: Bidders may have incentives to hide their demand for items in order to achieve a better price

Example of Demand Reduction in UPA


Real profile


 (1, 1, 1)


 (2/3, 0, 0)

 (1/2, 0, 0)

Equilibrium profile

 (1, 0, 0)

 (2/3, 0, 0)

 (1/2, 0, 0)

$OPT = 3, SW(\mathbf{b}) = 13/6 \Rightarrow PoA \geq 18/13$ for UPA

- Revealing the true profile for bidder 1 results in a relatively high price
- Demand reduction discussed further in [Ausubel, Cramton '96]

PoA for pure equilibria

Can demand reduction create a huge loss of efficiency?

Theorem:

For the Discriminatory Price Auction (DPA), and **arbitrary** monotone valuations for the bidders, **PoA = 1**

- No need to assume no-overbidding
- All pure Nash equilibria (when they exist) are efficient
- Generalizes what holds for the single-item 1st price auction
- Existence of pure equilibria guaranteed under appropriate tie-breaking rules

PoA for pure equilibria

- The same is not true for UPA
- **Example:** Consider k units and the profiles:

Real profile



$(k, 0, 0, \dots, 0)$



$(1, 1, 1, \dots, 1)$

Equilibrium profile **b**



$(1, 1, 1, \dots, 1)$



$(0, 0, 0, \dots, 0)$

- $OPT = 2k-1$
- $SW(\mathbf{b}) = k$
- $PoA \geq (2k-1)/k = 2 - 1/k$ for UPA
- Can it get worse?

PoA for pure equilibria

- For non-conservative bidders, it can get unbounded
- The no-overbidding assumption in UPA:

$$\sum_{j=1}^s b_i(j) \leq v_i(s) \quad \forall i, \forall s \leq k$$

[Birmpas, Markakis, Telelis, Tsikiris '17]:

For the Uniform Price Auction (UPA), and for

- Submodular bidders
- No-overbidding pure equilibria,

$$\text{PoA} \leq 2.18$$

- Tight example even for 2 bidders

PoA for mixed equilibria

[de Keijzer, Markakis, Schaefer, Telelis '13]:

For submodular valuations, the PoA for mixed equilibria is

- $\leq e/e-1$ for DPA
- $\leq 3.146 < 2e/e-1$ for UPA

Remarks:

- $3.146\dots = |W_{-1}(-1/e^2)|$ (Lambert W function)
- Bounds hold both for standard bidding and for the simplified uniform bidding format
- The same bounds also hold for Bayesian games (PoA for Bayes-Nash equilibria)

PoA for mixed equilibria

- Currently known lower bounds: ≈ 1.1 for DPA, **2.18** for UPA
 - Far from tight in the case of mixed equilibria
- Our proof can be cast into the smoothness framework of **[Syrgkanis, Tardos '13]**



- Upper bounds carry over to simultaneous and sequential compositions of multi-unit auctions (e.g. combinatorial multi-unit auctions)
- Similar approaches and techniques used in other types of auctions as well (e.g. item-bidding auctions)
[Christodoulou, Kovacs, Schapira '08, Bhawalkar, Roughgarden '11, Feldman, Fu, Gravin, Lucier '13]

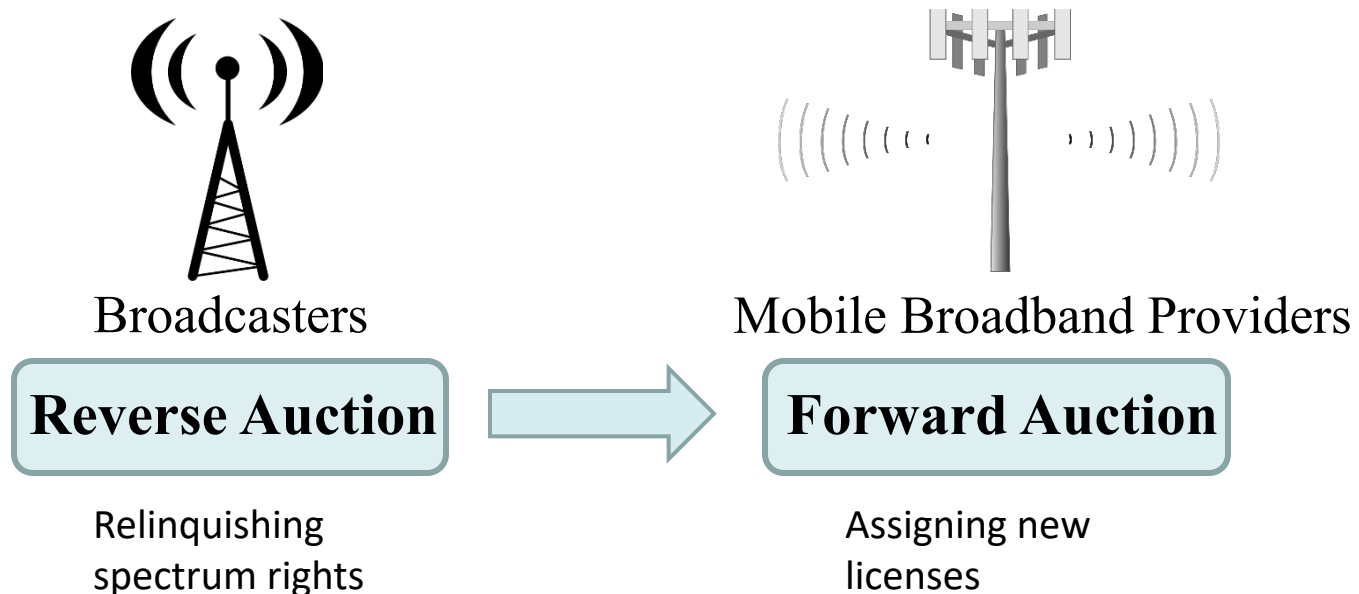
Conclusions on PoA

- **Take-home story:** simple auction formats used in practice perform quite well w.r.t. social welfare
- Upper bounds:
 - For pure equilibria, almost tight for sponsored search, completely tight for multi-unit auctions
 - Open if we can improve the bounds for mixed equilibria
 - PoA can also become even better if we focus on Nash equilibria in undominated strategies
- Lower bounds:
 - Much harder to get

Examples of truthful auctions in practice

Spectrum Auctions

- Deferred Acceptance Auctions initiated by [Milgrom, Segal '14]
- Motivated by the design of the FCC “**Broadcast Incentive Auction**”



- Commenced on March 2016, closed on April 2017 for repurposing spectrum to align with consumer demand for broadband services

Basic Mechanism Design Setting

Main features:

- A **provider** of some service or resources
- A set of **single-parameter buyers** $N = \{1, 2, \dots, n\}$ interested in (some of) the resources
- Each buyer has a **valuation** v_i
- For each buyer: need to make an **accept/reject** decision
- **Feasible solutions:** Only specific subsets of buyers may be served simultaneously, due to problem constraints (e.g. interference constraints in spectrum auctions)

The framework of Deferred-Acceptance Auctions

- Backward greedy allocation algorithms
- They work in rounds, finalizing the decision for a single bidder in each round
- A_t = set of active bidders at round t
- Score of bidder i at round t : $\sigma_i^{A_t}(b_i, b_{N \setminus A_t})$
 - non-decreasing in b_i
 - Possible dependence on the set A_t (but not on the bids of active bidders)

1. Initially all bidders are **active** ($A_1=N$)
2. While accepting all active bidders in A_t is **infeasible**
 - **Reject** the bidder i with the lowest score
 - $A_{t+1} = A_t \setminus \{i\}$
3. Remaining bidders are **accepted** and pay **threshold prices**

Properties of Deferred-Acceptance Auctions

Incentive guarantees:

- Not hard to show that DA auctions are truthful
- In fact we can have much stronger incentive guarantees

Definition: A mechanism is **weakly group-strategyproof** if: for any coalition $S \subseteq N$, and any profile b_{-S} , there is no deviation by S , such that all members are strictly better off, i.e., such that:

$$u_i(b_S, b_{-S}) > u_i(v_S, b_{-S}), \text{ for every } i \in S$$

Lemma: DA auctions are weakly group-strategyproof

Properties of Deferred-Acceptance Auctions

Further advantages of DA auctions:

1. Practical and simple to implement **as long as**
 - Scoring function is simple
 - Checking feasibility of a solution is easy
2. They admit an implementation as an ascending clock auction
3. Using the ascending auction implementation:
 - Very easy to argue that truth-telling is a dominant strategy (**obvious strategyproofness [Li '15]**)
 - Privacy preservation: winners do not reveal their true value

Possible limitations:

1. They do not always guarantee a good approximation to the social welfare
2. Same for other objectives (e.g. revenue)
3. Solution returned may not be a maximal set w.r.t. problem constraints (drawback of backward greedy algorithms)

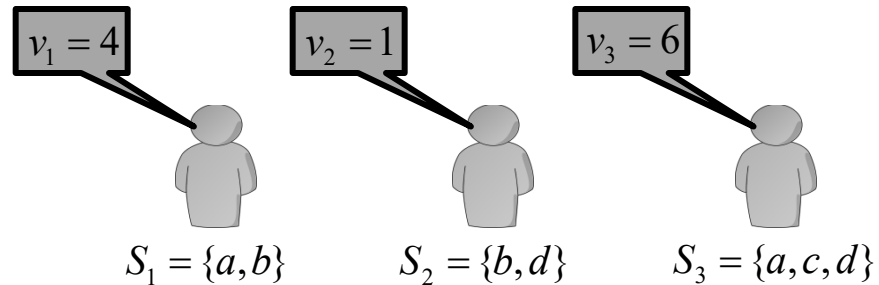
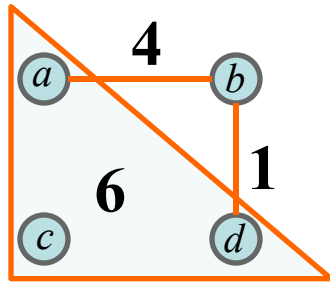
An illustration

Recall single-minded bidders from previous lectures

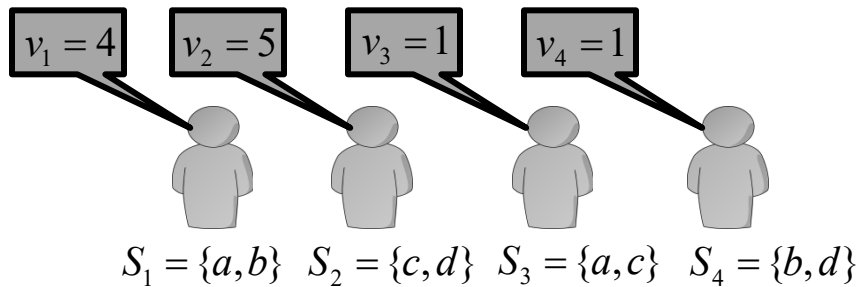
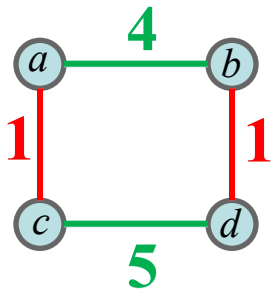
- The auctioneer has a set M of items for sale
- Each bidder i is interested in acquiring a specific subset of items, $S_i \subseteq M$ (known to the mechanism)
 - If the bidder does not obtain S_i (or a superset of it), his value is 0
- Each bidder submits a bid b_i for his value if he obtains the set
- Motivated by certain spectrum auctions
- Feasible allocations: the auctioneer needs to select winners who do not have overlapping sets

Single-minded bidders

Examples



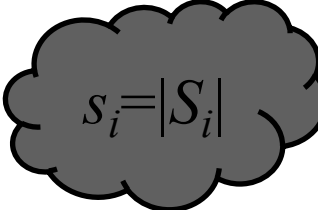
- In the example above, the auctioneer can accept only 1 bidder as a winner
- In the example below, the auctioneer can accept up to 2 bidders as winners



A forward greedy algorithm for single-minded bidders

[Lehmann, O' Callaghan, Shoham '01]:

- Order the bidders in decreasing order of $b_i/\sqrt{s_i}$
- Accept each bidder in this order unless overlapping with previously accepted bidders


$$s_i = |S_i|$$

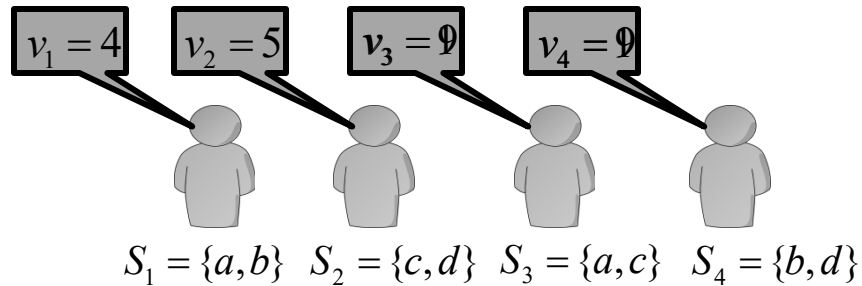
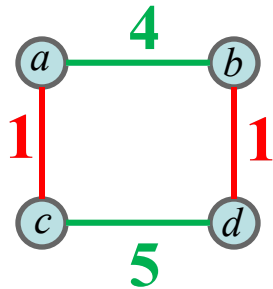
This algorithm achieves

- Monotonicity of the allocation (hence can be made truthful)
- $1/\sqrt{m}$ -approximation, where $m = |M|$
- $1/d$ -approximation, where $d = \max_i s_i$

Final conclusion: truthful polynomial time mechanism with the best possible approximation to the social welfare

Coalitions under the forward greedy mechanism

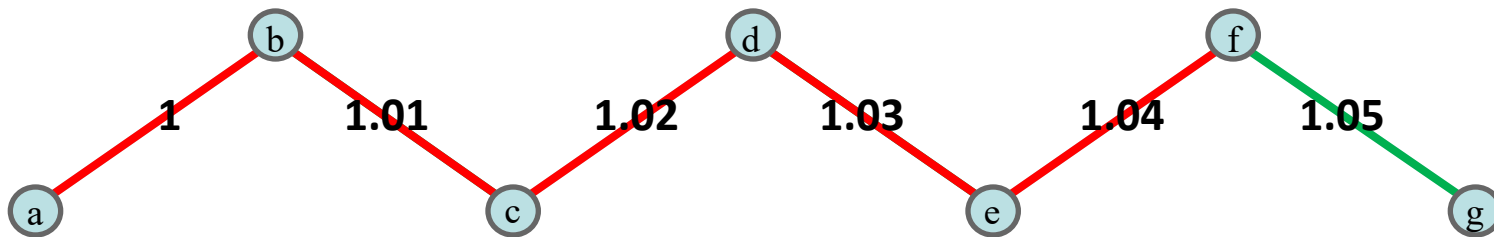
- The forward greedy mechanism is truthful but suppose players could also collude:



- What would forward greedy do?
 - Accept bid $\{c,d\}$
 - Reject bids $\{a,c\}$ and $\{b,d\}$
 - Accept bid $\{a,b\}$
 - Threshold price = 0
- The coalition $\{3, 4\}$ can change the outcome
- Threshold price still 0
- Both members better off!
- Forward greedy is not group-strategyproof

Scoring Functions for DA auctions

- Can we achieve similar welfare guarantees with backward greedy algorithms?
- How about a DA auction with scoring $\sigma_i(v_i, s_i) = v_i / \sqrt{s_i}$?



- Backward greedy can do much worse than forward greedy
- Use conflict number $\sigma_i(v_i, c_{i,t}) = v_i / c_{i,t}$?
 - $c_{i,t}$ = number of conflicts with other bidders at stage t

[Dutting, Gkatzelis, Roughgarden '14]:

- This does not work either
- Having s_i or $c_{i,t}$ in the denominator, raised to any power cannot achieve an $O(1/d)$ or $\tilde{O}(1/\sqrt{m})$ approximation

Positive results for DA auctions

[Dutting, Gkatzelis, Roughgarden '14]:

Theorem 1: There exists a DA auction that achieves an approximation ratio of $O(d)$

Theorem 2: There exists a DA auction that achieves an approximation ratio of $O(\sqrt{m} \log m)$

Main message:

We can have comparable approximations as in forward greedy, but with stronger incentive guarantees!

- And with a more complicated scoring function

Final conclusions

- A wide range of applications
- The full spectrum of incentive guarantees can be seen in practice
 - **Non-truthful and bad equilibria** (uniform price auction or sponsored search with overbidding)
 - **Non-truthful and efficient equilibria** (single-item first price auction)
 - **Non-truthful and relatively efficient equilibria** (sponsored search, uniform price auction, under no-overbidding)
 - **Truthful** (single-item Vickrey)
 - **Weakly group-strategyproof** (DA auctions)
- The choice of mechanism deployed may depend on:
 - Traditions and practices used in a specific application domain (not always easy to switch to a new format)
 - Complexity considerations (simplicity is often a must)
 - Legal issues (there exist governmental auctions where social welfare w.r.t. reported bids needs to be maximized)