

Άσκηση 8, φυλλάδιο "ΑΣΚ. Θ. GAUSS"

Ποή του $\vec{F} = f \nabla f$ διαφέρει του S (τόση Θ Gauss) είναι

$$\iiint_K \operatorname{div} \vec{F}.$$

$$\nabla f = (f_x, f_y, f_z) \Rightarrow \vec{F} = (f f_x, f f_y, f f_z)$$

$$\operatorname{div} \vec{F} = \frac{\partial}{\partial x} (f f_x) + \frac{\partial}{\partial y} (f f_y) + \frac{\partial}{\partial z} (f f_z)$$

$$= f_x^2 + \underbrace{f_{xx}} + f_y^2 + \underbrace{f_{yy}} + f_z^2 + \underbrace{f_{zz}} = |\nabla f|^2.$$

Άσκηση: Διvekou το διαv. πεδίο

$$\vec{F} = (x^2 e^z + x, y - x e^z, x^2 - z x e^z).$$

Να βρείτε τη ποσότητα $\iint_S \vec{F} \cdot d\vec{S}$ διαφέρουσα την

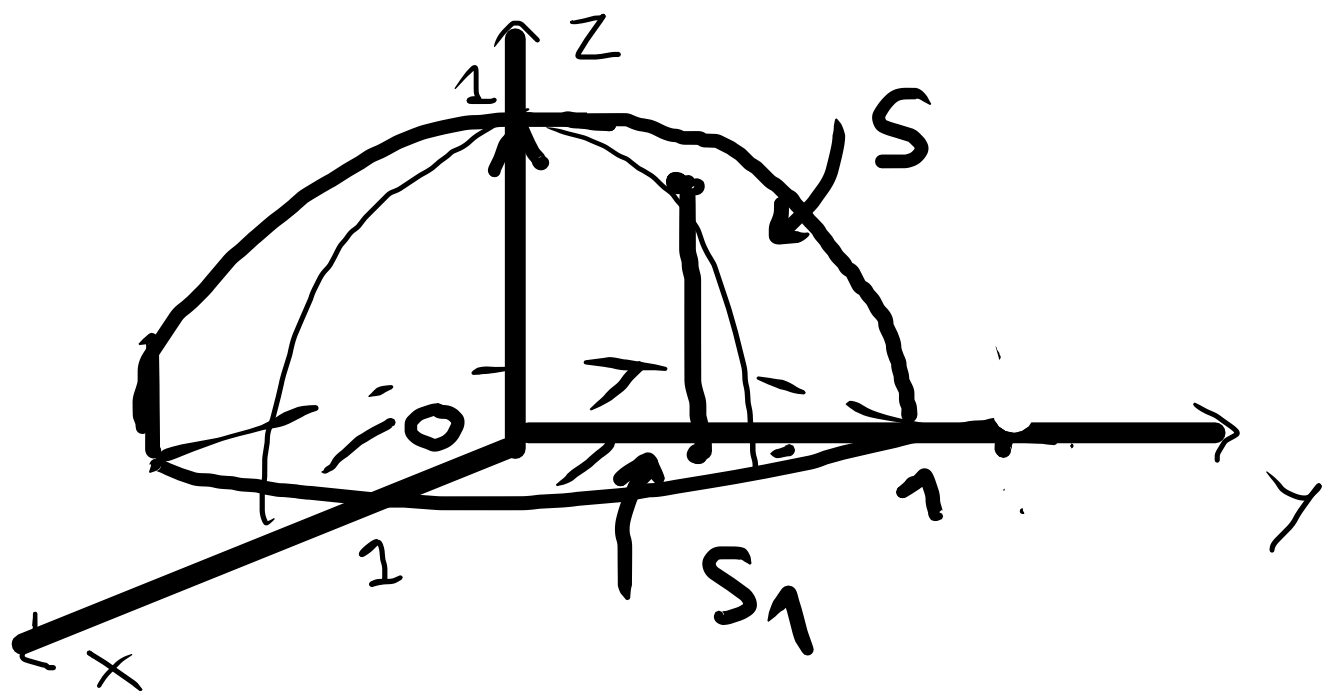
$$S = \{ (x, y, z) : x^2 + y^2 + z^2 = 1, z \geq 0 \}$$

$$\text{Θέω } S_2^* = S^* \cup S_1^*,$$

$$S_1^*: x^2 + y^2 \leq 1, z = 0$$

ποση διαφέρουσα την $S =$

$$= \text{ποη διαφ.}(S_2) - \text{ποη διαφ.}(S_1)$$



$$S_2 = \underbrace{\lambda \epsilon_0 \epsilon_n}_0 \cdot \text{Gauss's point} \cdot \text{data} \cdot (S_2) =$$

$$= \iiint_K \text{div } \vec{F}$$

$$K = \{ (x, y, z) : x^2 + y^2 + z^2 \leq 1 \}$$

$$= \iiint_K (2x^2 + 1 + 1 - 2x^2) dx dy dz =$$

$$= 2 \iiint_K dx dy dz = 2 \iint_{x^2 + y^2 \leq 1} \left(\int_0^{\sqrt{1-x^2-y^2}} dz \right) dx dy$$

$$= 2 \iint_{x^2+y^2 \leq 1} \sqrt{1-x^2-y^2} \, dx \, dy \quad \stackrel{\text{polar}}{=} \quad 2 \int_0^{2\pi} \int_0^1 \sqrt{1-r^2} \, r \, dr \, d\phi$$

$$= 4\pi \int_0^1 \sqrt{1-r^2} \, r \, dr \quad \stackrel{t=1-r^2}{=} \quad 2\pi \int_0^1 \sqrt{t} \, dt$$

$$= 2\pi \int_0^1 \sqrt{t} \, dt = 2\pi \cdot \frac{2}{3} t^{3/2} \Big|_0^1 = \boxed{4\pi/3}$$

$$S_1: \Delta \rightarrow \mathbb{R}^3, \quad S_1(u, v) = (u, v, 0), \quad \Delta = \{(u, v) : u^2 + v^2 \leq 1\}$$

$$\frac{\partial S_1}{\partial u} \times \frac{\partial S_1}{\partial v} = (0, 0, 1) \quad \vec{n}(u, v) = (0, 0, -1)$$

$$\text{Flux of } \vec{F} \text{ through } (S_1) = \iint_{\Delta} \vec{F}(S_1(u, v)) \cdot \vec{n}(u, v) du dv$$

$$= \iint_{\Delta} (u^2 + u, v - u, u^2 - 2u) \cdot (0, 0, -1) du dv$$

$$= \iint_{\Delta} (2u - u^2) du dv = 2 \int_0^{2\pi} \int_0^1 r^2 \cos \varphi dr d\varphi -$$

$$- \int_0^{2\pi} \int_0^1 r^3 \cos^2 \varphi dr d\varphi = -\frac{1}{4} \int_0^{2\pi} \cos^2 \varphi d\varphi =$$

$$= \dots = \underline{\underline{-\pi/4}}$$

τε > 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52, 55, 58, 61, 64, 67, 70, 73, 76, 79, 82, 85, 88, 91, 94, 97, 100, 103, 106, 109, 112, 115, 118, 121, 124, 127, 130, 133, 136, 139, 142, 145, 148, 151, 154, 157, 160, 163, 166, 169, 172, 175, 178, 181, 184, 187, 190, 193, 196, 199, 202, 205, 208, 211, 214, 217, 220, 223, 226, 229, 232, 235, 238, 241, 244, 247, 250, 253, 256, 259, 262, 265, 268, 271, 274, 277, 280, 283, 286, 289, 292, 295, 298, 301, 304, 307, 310, 313, 316, 319, 322, 325, 328, 331, 334, 337, 340, 343, 346, 349, 352, 355, 358, 361, 364, 367, 370, 373, 376, 379, 382, 385, 388, 391, 394, 397, 400, 403, 406, 409, 412, 415, 418, 421, 424, 427, 430, 433, 436, 439, 442, 445, 448, 451, 454, 457, 460, 463, 466, 469, 472, 475, 478, 481, 484, 487, 490, 493, 496, 499, 502, 505, 508, 511, 514, 517, 520, 523, 526, 529, 532, 535, 538, 541, 544, 547, 550, 553, 556, 559, 562, 565, 568, 571, 574, 577, 580, 583, 586, 589, 592, 595, 598, 601, 604, 607, 610, 613, 616, 619, 622, 625, 628, 631, 634, 637, 640, 643, 646, 649, 652, 655, 658, 661, 664, 667, 670, 673, 676, 679, 682, 685, 688, 691, 694, 697, 700, 703, 706, 709, 712, 715, 718, 721, 724, 727, 730, 733, 736, 739, 742, 745, 748, 751, 754, 757, 760, 763, 766, 769, 772, 775, 778, 781, 784, 787, 790, 793, 796, 799, 802, 805, 808, 811, 814, 817, 820, 823, 826, 829, 832, 835, 838, 841, 844, 847, 850, 853, 856, 859, 862, 865, 868, 871, 874, 877, 880, 883, 886, 889, 892, 895, 898, 901, 904, 907, 910, 913, 916, 919, 922, 925, 928, 931, 934, 937, 940, 943, 946, 949, 952, 955, 958, 961, 964, 967, 970, 973, 976, 979, 982, 985, 988, 991, 994, 997, 1000

$$= \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$